What is Good Mathematics Teaching?
(An Open Letter to My Preservice Teachers)

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As a mathematics educator and a teacher of mathematics content courses, I think often about this question, though I have seldom, if ever, committed my thoughts to writing. In my current role as supervisor of student teachers at X University, this seems an ideal time to address the subject of high-quality mathematics teaching.

The following comments come from 28 years of teaching high school mathematics (first-hand experience), several years of graduate school (theoretical knowledge and research), and several years of working with student teachers and preservice teachers in methods classes and in the field (practical experience). These are ***my*** beliefs, although I think you will find that my beliefs align closely with those of the National Council of Teachers of Mathematics (NCTM) and the Association of Mathematics Educators (AMTE), two of the foremost organizations dedicated to the teaching of mathematics and the preparation of future mathematics teachers, respectively.

When I was teaching high school mathematics, I didn't have a name for what I was doing or how I liked for my students to learn, but it felt right to me and usually produced good results. For me it was always critical that students understand ***why*** as well as ***what*** to do, because real learning only occurs when students make sense of what they are asked to learn. In graduate school I learned that I am a ***constructivist***, because I believe that students must build their understanding of mathematics for themselves by making connections between prior knowledge and new knowledge.

I always knew that there were two types of learning in mathematics as well—understanding the concepts and knowing how to do the procedures. Now I know that mathematics educators refer to these two types of understanding as ***conceptual understanding*** and ***procedural competence*** (National Research Council, 2001). In simple terms, for meaningful learning to take place, students must understand both why and how; one type of learning without the other is insufficient for students to successfully build their own sense of what they are learning.

As a result, I learned to avoid "rules" and shortcuts without meaning because I knew that my students would eventually become confused or forget the cute tricks that might work today but have no chance for long-term recall. That doesn't mean that my students never used shortcuts or acronyms, because they did. My students learned to "FOIL" but not without first understanding the overarching principle of how the distributive property applies to multiplication of polynomials in general. My geometry students would remember "SOH-CAH-TOA" as a way of defining the three basic trigonometric ratios, but they also understood how to create the graph of the sine function without necessarily having to plug the function into the calculator.

Speaking of the calculator and technological tools in general, I am a huge proponent of the ***appropriate*** use of any and all such tools. In fact, while I was in graduate school I worked for the Center for Technology and Teacher Education in the Curry School of Education at the University of Virginia. Although I was already relatively proficient in the use of most common technologies (graphing calculator, Excel, and the Geometer's Sketchpad), I learned more about how to integrate them into instruction both as a demonstration tool and as tools for student use. I also learned more about how to use interactive white boards for mathematics instruction, and I was part of a research study into the effectiveness of such boards and other instructional technology on student understanding of mathematics.

The UVA Center for Technology and Teacher Education has formulated a set of guidelines for the use of technology in the teaching of mathematics (Garofalo, Drier, Harper, Timmerman, & Shockey, 2000). The five guidelines are the following:

1. Introduce technology in context
2. Address worthwhile mathematics with appropriate pedagogy
3. Take advantage of technology
4. Connect mathematics topics
5. Incorporate multiple representations

Their website (http://www.teacherlink.org/content/math/guidelines.html) provides more information about each of the five guidelines and is something with which every mathematics teacher should be familiar. I recommend you take a look.

Next, I would like to address the model of instruction that you choose to utilize and lobby for you to strongly consider an inquiry-based model whenever possible. While many think that inquiry-based learning is appropriate only for science classes, teachers of mathematics can also integrate this style of learning very effectively. For example, suppose we were teaching a lesson on multiplication of monomials in which the goal was for students to know that the product of x^2 and x^3 was x^5, obtained by adding the exponents. The direct instructional approach would be to state the rule and have students practice applying the rule. The inquiry-based approach would provide students with examples that would lead them to discover their own rule. They would write x^2 as $x * x$ and x^3 as $x * x * x$ so that the product would be $x * x * x * x * x$. Counting the number of identical factors and applying their prior knowledge of the meaning of an exponent, they would rewrite the expression as x^5. Several examples would lead them to conclude that it would be more efficient, yet still accurate, to add the exponents. They would then have discovered their own “rule” and would have ownership of that knowledge, since they would have created it. Contrast that with the scenario in which the teacher dictates or writes a rule and asks students to accept and apply it.

I can still remember students and classes I taught where they would beg me to just “tell us what to do.” The topic is irrelevant—the point is that I would insist that whatever we were learning had to come from the students—with my guidance, of course. My role was that of facilitator, rather than disseminator of information, and students had a hard time with this approach at first, since they were accustomed to the teacher telling them everything they had to know. Not only would I insist that the students generate the knowledge—I would require them to justify themselves and explain “why” something was true.

While this was painful at first, I knew that I had won the battle when a student answered a question in class one day and then said, “I know you’re going to ask me ‘why,’ so I’ll just go ahead and tell you now” without being prompted. That student finally ***got it*** because I had the expectation that students would be able to solve problems and justify their solutions. I set the bar high and kept it there, but I also made sure that students had the help they needed to reach that level of proficiency. Rarely did I worry about the SOLs, either, because I was confident that my students would achieve well beyond those minimal standards, and year after year the results bore this out.

Consider, too, the role of discrepant events in your teaching of common mathematical concepts. To use the previous example as an illustration, it might seem cruel to do this, but what would happen if you asked students to simplify $(x^2)^3$ in the midst of a lesson on adding exponents? In my experience I found that students would nearly always arrive at x^5 as their answer, because they would argue that we were adding exponents today. This type of discrepant event, an example that contradicts or extends what they are primarily focused on, creates a state of “conceptual disequilibrium” that causes students to think about and rethink the knowledge they are constructing for themselves (Longfield, 2009). In our quest to simplify the mathematics we teach to the point where students cannot possibly become confused, we may be doing them a disservice. If they never have to grapple with the contrast between these two examples (not to mention $x^2 + x^3$), then they never have an opportunity to build the understanding that is so important. Introducing discrepant events helps to eliminate common misconceptions by bringing them out into the open where they can be identified, discussed, and corrected.

I apologize for the length of this treatise on good mathematics teaching, but I did want to give you some food for thought. One of the reasons I went back to graduate school at a ripe old age was to get to where I am today—teaching at the college level and working with preservice teachers like you. My experience in having done what you are now doing is something that I will readily share with you. During my years as a classroom teacher, I have made just about every error that can be made, and I have learned from my mistakes and experiences. I am only too happy to be able to “talk math” with you and help devise effective ways to teach whatever it is you are asked to teach. I am here for you and stand ready to support you throughout this experience.

References

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